



MODELING OF THE STATE OF THE CONTINUOUS ENVIRONMENT

Educational component work program (Syllabus)

Educational component details	
Level of higher education	<i>Third (educational and scientific)</i>
Branch of knowledge	<i>13 Mechanical engineering</i>
Specialty	<i>131 Applied Mechanics</i>
Educational program	APPLIED MECHANICS
Status of the educational component	<i>Selective</i>
Form of study	<i>full-time (day)</i>
Year of preparation, semester	<i>2nd year, spring semester</i>
The scope of educational component	5(150)
Semester control / control measures	<i>Exam</i>
Lessons schedule	
Language of instruction	<i>Ukrainian</i>
Information about course leader / teachers	Lecturer: https://cpsm.kpi.ua/karvatskij-anton-yanovich.html Practical: https://cpsm.kpi.ua/karvatskij-anton-yanovich.html Laboratory: –
Course placement	Link to remote resource Google classroom, https://campus.kpi.ua/tutor/index.php?mode=mob&show&irid=261222 https://cpsm.kpi.ua/navchannya/sylabusy/sylabusy-2024-roku.html

Educational component program

1. Description of the educational component, its purpose, subject of study and learning results

At the moment, it is difficult to imagine a thesis on technical sciences, in which there was no section devoted to the numerical analysis of the physical fields of industrial equipment in the approximation of a solid environment. That is why graduate students of the specialty Applied Mechanics need to have the skills of modeling the state of a solid environment using modern CAD-CAE-systems, which is necessary for the development of new and modernization of existing industrial equipment and, in particular, materials and containers for packaging and packaging equipment.

The purpose of the educational component is to form and develop the following competencies (abilities) of PhD applicants: integral, general, professional and program learning outcomes.

The subject of the educational component "Modeling of the state of a continuous environment" is a simulation of the stress-deformed state of solid, liquid and gaseous bodies in their interaction with each other and physical fields of different physical nature – gravitational, thermal, electromagnetic, radiation, etc.

The study of the educational component enables graduate students to conduct theoretical research of processes and equipment using modern CAD-CAE systems.

In accordance with the goal, the training of masters in this specialty requires strengthening the formation of the following competencies:

1.1 Integral: *The ability to solve complex problems in the field of applied mechanics, including research and innovation, which involves a deep rethinking of existing and the creation of new holistic knowledge and / or professional practice.*

1.2. General:

- ZK 1 Ability to identify and solve problems.
- ZK 2 Ability to make informed decisions.
- ZK 3 Ability to generate new ideas (creativity).
- ZK 4 Ability to develop and manage projects.

1.3. Professional:

- FC 1 The ability to critically analyze, evaluate and synthesize new and complex ideas in the process of researching mechanical structures, machinery, materials and production processes of mechanical engineering based on the latest knowledge in the field of mechanics and related subject industries.
- FC 4 Ability to critically understand problems in teaching, professional and research activities at the level of the latest achievements of engineering sciences and on the verge of subject areas.
- FC 5 Ability to set a problem and determine ways to solve the problem by means of applied mechanics and related subject areas, knowledge of methods for finding the optimal solution under conditions of incomplete information and conflicting requirements.

1.4. Programmatic learning outcomes:

- PH 1 Know the general theory and methods of conducting scientific research and be able to practically apply them for research of objects in the field of mechanical engineering.
- PH 2 Perform a scientific search and, based on the analysis of its results, determine the ways to solve the tasks.
- PH 8 Skills in the use of modern computer tools and information technologies in scientific activities, in particular in the implementation of experimental research.

2. Prerequisite and requisition educational components (place in the structural and logical scheme of training according to the relevant educational program)

Prerequisite: *the educational component is based on the study of the following credit modules: Methods of design and calculation of machines and structures, reliability of machines and structures.*

Post requisition: *Modeling of technical objects and systems, Perspective directions of development of energy and resource efficient processes, equipment and technologies.*

3. Contents of the educational component

Section 1. Basics of solid media mechanics

- Topic 1.1. Basics of tensor calculus
- Topic 1.2. Tense state
- Topic 1.3. Deformed state
- Topic 1.4. Movement and flow
- Topic 1.5. Laws of solid environment mechanics
- Topic 1.6. Linear elasticity theory
- Topic 1.7. Mechanics of liquids and gases
- Topic 1.8. Theory of Plasticity
- Topic 1.9. Linear viscosity
- Topic 1.10. Turbulent movement of a solid medium

Section 2. Applied nonlinear problems of solid media mechanics

- Topic 2.1. Mathematical formulation of nonlinear problems of mechanics continuum (MC)

Topic 2.2. Numerical methods for solving nonlinear MC problems based on the finite element method

Topic 2.3. Numerical methods for solving nonlinear problems of the heat-hydrodynamic state of Newtonian and non-Newtonian fluids on the basis of the finite volume method

Topic 2.4. Software implementation of numerical methods for solving nonlinear PROBLEMS MC

4. Educational materials and resources

Basic literature

1. Karvatskii A. Y. *Mechanics of continuum: teaching. posib.* Kyiv: KPI them. Igor Sikorsky Kyiv Polytechnic Institute, 2017. 292 p. Vulture – recommended by the Academic Council of NTUU "KPI" (Protocol No. 11 of 07.11.2016) URL : <http://ela.kpi.ua/handle/123456789/18094>

2. Karvatskii A.Y. *Method of finite elements in problems of mechanics of continuum. Software implementation and visualization of results [Text]: teaching. posib.* Kyiv: NTUU "KPI" VPI "Polytechnic", 2015. 392 p. Stamp provided by the Academic Council of NTUU "KPI" (Protocol No. 4 of 12.05.2015) URL : <http://ela.kpi.ua/handle/123456789/11636>

3. *Mechanics of continuum. Theoretical foundations of the discipline [Electronic resource]: teaching. posib. for stud. specialties 131 "Applied Mechanics", 133 "Branch Engineering", specialization "Engineering, computer modeling and design of packaging equipment", "Engineering, computer modeling and design of equipment for the production of polymeric and building materials and products" / A. Y. Karvatskii.* Kyiv: Igor Sikorsky Kyiv Polytechnic Institute, 2018. 290 p. Vulture provided by the Methodical Council of Igor Sikorsky Kyiv Polytechnic Institute (Protocol No. 10 of 21.06.2018) URL : <http://ela.kpi.ua/handle/123456789/23917>

4. Karvatskii A.Y. *Modeling of energy-saving regulations of industrial equipment: teaching. posib.* Kyiv: NTUU "KPI", 2014. 234 p. Stamp provided by the Ministry of Education and Science of Ukraine (Letter No. 1/11-2299 of 11.02.2014) URL : <https://cpsm.kpi.ua/publikatsiji/kniqi/956-modelyuvannya-energozberigayuchikh-reqlamentiv-promislovoogo-obladnannya.html>

5. Karvatskii A. Y. *Nonlinear problems of mechanics of solid media. Computer workshop in the discipline [Electronic resource]: teaching. posib. for stud. specialties 133 "Branch engineering", specialization "Engineering, computer modeling and design of equipment for the production of polymeric and building materials and products".* Igor Sikorsky Kyiv Polytechnic Institute. – Electronic text data (1 file: 2,480 MB). Kyiv: Igor Sikorsky Kyiv Polytechnic Institute, 2019. 158 p. URL : <http://ela.kpi.ua/handle/123456789/27528>

Additional literature

1. Karvatskii A. Y. *Mechanics of continuum. Problem solving: teaching. posib.* Kyiv: NTUU "KPI", 2016. 391 p. Vulture provided by the Academic Council of NTUU "KPI" (Protocol No. 5 of 11.04.2016) URL : <http://ela.kpi.ua/handle/123456789/15428>

2. Mase G. E. *Theory and Problems of Continuum Mechanics.* New York : McGraw-Hill Book Company. 1970. 230 p. URL : <https://www.twirpx.com/file/897819/>

3. *Mechanics of continuum – 1. Mechanics of continuum in engineering calculations: Text of lectures for students of specialties "Equipment of chemical industries and enterprises of building materials" / Compiled by O.S. Sakharov, A.Y. Karvatsky* Kyiv: NTUU "KPI", 2013. 231 p. (Certificate of electronic publication of IHF No. X 10/13-88). Electronic text data (1 file: 1.55 MB). Kyiv: NTUU "KPI", 2013. – 231 p. URL : <https://cpsm.kpi.ua/Doc/MSS-1.pdf>

4. *Mechanics of continuum – 2. Nonlinear problems of the mechanics of continuum. Lecture notes on the discipline [Electronic resource]: teaching. posib. for stud. specialties 133 "Branch engineering", specialization "Engineering, computer modeling and design of equipment for the production of polymeric and building materials and products".* Kyiv: Igor Sikorsky Kyiv Polytechnic Institute, 2018. 94 p. Vulture provided by the Methodical Council of Igor Sikorsky Kyiv Polytechnic Institute (Protocol No. 10 of 21.06.2018) URL : <http://ela.kpi.ua/handle/123456789/23918>

5. Zienkiewicz, O., Taylor, R., Fox, D.: *The Finite Element Method for Solid and Structural Mechanics*. 7th edn. Elsevier Ltd., Oxford (2014).
6. Segerlind Larry J. *Applied Finite Element Analysis* ; sec. edit. New York : John Wiley & Sons, Inc., 1984. 393 p. URL : <https://www.scribd.com/doc/241729488/Applied-Finite-Element-Analysis-Larry-J-Segerlind-pdf>
7. Poinso T., Veynante D. *Theoretical and numerical combustion*. 2nd ed. Philadelphia : Edwards, 2005. 522 p.
8. Thompson, M. *ANSYS Mechanical APDL for Finite Element Analysis*. 1st ed. Oxford: Butterworth-Heinemann, 2017. 462 p.
9. *Energy-resource-efficient production of isostatic graphite: monograph* / E. M. Panov, S. V. Leleka, A. Ya. Karvatskii, I. O. Mikulonok. Kyiv: Igor Sikorsky Kyiv Polytechnic Institute, 2020. 140 c. ISBN 978-617-7734-26-9 URL : <https://ela.kpi.ua/handle/123456789/37590>
10. *Energy-resource-efficient pressing of carbon graphite products: monograph* / E. M. Panov, S. V. Leleka, A. Ya. Karvatskyi, I. O. Mikulonok. Kyiv: Igor Sikorsky Kyiv Polytechnic Institute, 2020. 84 c. ISBN 978-617-7734-27-6 URL : <https://ela.kpi.ua/handle/123456789/37591>
11. Website: OpenFOAM. Open Source Field Operation And Manipulation CFD ToolBox – an open integrated platform for numerical simulation of solid media mechanics problems URL: <http://www.openfoam.com/>
12. Website: LIGGGHTS Open Source Discrete Element Method Particle Simulation Code. URL: <https://www.cfdem.com/>
13. Website: ParaView. An open-source, multi-platform data analysis and visualization application. URL: <https://www.paraview.org/>
14. *Modeling of statics and dynamics of bulk materials in LIGGGHTS [Electronic resource]: teaching. posib. for applicants for master's degree in specialties 131 Applied Mechanics, 133 Branch Engineering / A.Y. Karvatskii, I.O. Mikulonok, V.M. Vytvytsky; Igor Sikorsky Kyiv Polytechnic Institute. – Electronic text data (1 file: 4.16 MB). – Kyiv: Igor Sikorsky Kyiv Polytechnic Institute, 2021. – 76 p. URL : <https://ela.kpi.ua/handle/123456789/45613>*
15. Website: ANSYS Engineering Simulation & 3D Design Software ANSYS [Electronic resource]. URL : <https://www.ansys.com/>

Information resources on the Internet

1. Information resources on the Internet, <http://ela.kpi.ua/handle/123456789/18094>, <http://ela.kpi.ua/handle/123456789/11636>, <http://ela.kpi.ua/handle/123456789/23917>, <http://ela.kpi.ua/handle/123456789/15428>
2. Website: Department of Chemical, Polymeric and Silicate Engineering, <http://cpsm.kpi.ua/mr.html>
3. Campus Igor Sikorsky Kyiv Polytechnic Institute, <http://login.kpi.ua/>
4. Distance learning platform "Sikorsky", <https://sikorsky-distance.kpi.ua/>

Educational content

5. Methods of mastering the educational component (educational component)

Lecture classes

No s/p	The name of the lecture topic and the list of main issues (links to literature and tasks on the IWS)	Number of hours
1	Topic 1.1. Basics of tensor calculus <i>Lecture 1. Strainers. The rank of tensor. Physical laws. Vectors and scalars. Coordinate systems. Basic vectors. Triedron of single vectors. Actions on vectors. Diads and diadicas. Index designations. The interval for changing</i>	2

No s/p	The name of the lecture topic and the list of main issues (links to literature and tasks on the IWS)	Number of hours
	<i>indexes and agreeing on the summation. Approval of summation in symbolic designations. Coordinate conversion. General concept of strain. Metric tensor. Cartesian strainers.</i> <u>Tasks on the IWS.</u> <i>Cartesian strainers. The laws of transformation of Cartesian tensors. Kroneker Delta. Conditions of orthogonality. Actions on tensors. Symmetry of dyadites, matrices and tensors. The main values and main directions of symmetric tensors of the second rank. Tensor fields. Differentiation of tensors. Curvilinear integrals. Stokes's theorem. Gauss-Ostrohradsky's theorem. Literature: [1]: section 1, § 1.1–1.23, pp. 11–52, [A1]: Annex A, § A.1, pp. 319–335; electronic lecture notes.</i>	
2	Topic 1.2. Tense state <u>Lecture 2.</u> <i>The concept of a solid environment. Homogeneity. Isotropics. Density. Massive forces. Surface forces. The principle of tension koshi. Vector of tension. Tense state at the point. Tensor tension. The relationship between the tensor of tension and the vector of tension. Balance of forces and moments. Symmetry of tension tensor. The laws of transformation of the tension tensor. Surfaces of cauchy tension. The main tensions. Tensor tension invariants. Ellipsoid of tension. Maximum and minimum tactile stress. Mora's circles for tension. Flat tense state. Deviator and bullet tensor tension.</i> <u>Tasks on the IWS.</u> <i>Topics 1.2, 1.3. Literature: [1]: section 2, § 2.1–2.14, cc. 53–78, section 3, § 3.1–3.16, 79–107; [A1]: Annex A, § A.2, pp. 335–344, § A.3, pp. 344–361; electronic lecture notes.</i>	1
3	Topic 1.4. Movement and flow <u>Lecture 3.</u> <i>Movement. Current. Material derivative. Speed. Acceleration. Instant speed field. Trajectory. Tok lines. Established movement. Deformation rate. Edging. Increase of deformation. Physical interpretation of tensors of the rate of deformation and edging. Material derivatives of the volume element, surface element and linear element. Material derivatives of the integral by volume, integral on the surface and linear integral.</i> <u>Tasks on the IWS.</u> <i>Topics 1.4, 1.5. Literature: [1]: section 4, § 4.1–4.7, 108–122, section 5, § 5.1–5.7, 123–134; [A1]: Annex A, § A.4, pp. 361–368, § A.5, pp. 368–370; electronic lecture notes.</i>	1
4	Topic 1.6. Linear elasticity theory <u>Lecture 4.</u> <i>Hooke's generalized law. The function of deformation energy. Isotropic and anisotropic media. Symmetry of elastic properties. Isotropic environment. Elastic steels. Setting static and dynamic problems of elasticity theory. The superposition theorem. The only solution. The Principle of Saint-Venan. Flat problems of elasticity theory. Flat tense state and flat deformation. The functions of Erie's tension. Two-dimensional static problems of elasticity theory in polar coordinates. Hyperelasticity. Hypoelastcity. Linear thermal elasticity.</i> <u>Tasks on the IWS.</u> <i>Topics 1.6. Literature: [1]: section 6, § 6.1–6.10, 136–154; [A1]: Annex A, § A.6, pp. 370–373; electronic lecture notes.</i>	1
5	Topic 1.7. Mechanics of liquids and gases <u>Lecture 5.</u> <i>Fluid pressure. Tensor viscous tensions. Barotropic current. Defining or physical equations. Runoff liquids. Newtonian fluids. The basic equations of Newtonian fluid. The Navier-Stokes-Duham equation. Established current. Hydrostatics. Dead-up current. Perfect liquid. Bernouli equation. Circulation. Potential current. Flat potential current.</i> <u>Tasks on the IWS.</u> <i>Topics 1.7. Literature: [1]: section 7, § 7.1–7.6, 155–165;</i>	1

No s/p	The name of the lecture topic and the list of main issues (links to literature and tasks on the IWS)	Number of hours
	<i>[A1]: Annex A, § A.7, pp. 373–377; electronic lecture notes.</i>	
6	Topic 1.8. Theory of Plasticity <u>Lecture 6.</u> Basic provisions and definitions. Idealized diagrams of plastic behavior. Conditions of plasticity. Criteria of Tresca and Mises. Space of tension. P-plane. Surface fluidity. Behavior of the material beyond fluidity. Isotropic and kinematic strengthening. The relationship between stresses and deformations in the plastic state. Theory of Plastic Potential. Equivalent tension. Equivalent increase in plastic deformation. Work on plastic deformations. Hypotheses of strengthening. The ratio of the incremental theory of plasticity for the environment with isotropic strengthening. Deformation theory of plasticity. Problems of elastic plasticity. <u>Tasks on the IWS.</u> Topics 1.8, 1.9. <u>Literature:</u> [1]: section 8, § 8.1–8.13, 172–198, section 9, § 9.1–9.8, 200–217; [A1]: Annex A, § A.8, pp. 377–379, § A.9, pp. 380–383; electronic lecture notes.	2
7	Topic 1.10. Turbulent movement of a solid medium <u>Lecture 7.</u> Classic turbulence models for Reynolds stresses. Prandtl model, built on an algebraic expression. Prandtl-Colmour model with one equation. isotropic turbulence model with two scalar equations. Mathematical models for numerical modeling of turbulent flows. RANS turbulence models. Les turbulence models. Hybrid models of turbulence DES. <u>Tasks on the IWS.</u> Topics 1.10. <u>Literature:</u> [A5]: lecture. 9, p. 80–86; [1]: section 11, § 11.1–11.4, 256–271; [4]: section 9, § 9.1–9.3, 198–218; electronic lecture notes.	2
8	Topic 2.1. Mathematical formulation of nonlinear problems MC <u>Lecture 8.</u> Nonstationary problem of thermal conductivity. Nonstationary problem of thermal conductivity. Elastic-plastic problem taking into account isotropic strengthening. The problem of elastic and plastic behavior of bulk materials. Problems of heat-hydrodynamic state of Newtonian fluid during laminar and turbulent motion. Basic equations for nenyuton fluid. Models of dilatant (composite) substances. Formulation of the problem of the heat-hydrodynamic state of the liquid type Bingham-Papanastasiou. <u>Tasks on the IWS.</u> The main system of differential equations MCC. Initial and Boundary Conditions. <u>Literature:</u> [5]: section 3, § 3.1–3.3, 32–42, section 4, § 4.1–4.3, 44–53; [1]: section 7, § 7.7–7.8, 166–171; electronic lecture notes.	2
9	Topic 2.2. Numerical methods for solving nonlinear MCC problems based on the finite element method <u>Lecture 9.</u> Methods of numerical solution of problems of thermal conductivity, thermal-electrical conductivity, elastic-plastic behavior of metals and bulk materials. Reverse display algorithm. <u>Tasks on the IWS.</u> <u>Literature:</u> [5]: chapter 1, § 1.1–1.3, p. 9–19, section 2, § 2.1–2.3, 20–30, section 3, § 3.1–3.3, 32–42, section 4, § 4.1–4.3, 44–53; [4]: section 7, § 7.1–7.2, 178–184; electronic lecture notes.	2
10	Topic 2.3. Numerical methods for solving nonlinear problems of the heat-hydrodynamic state of Newtonian and non-Newtonian fluids on the basis of the finite volume method <u>Lecture 10.</u> Numerical methods for solving problems of laminar and turbulent motion of Newtonian incompressible fluid. Simple algorithm. Numerical methods for solving problems of laminar movement of Nenuton liquids. <u>Tasks on the IWS.</u> Formulation of initial and boundary conditions. <u>Literature:</u> [4]: section 5, § 5.1–5.2, 127–139, section 9, § 9.1–9.3, 198–218; electronic lecture notes.	2

No s/p	The name of the lecture topic and the list of main issues (links to literature and tasks on the IWS)	Number of hours
11	Topic 2.4. Software implementation of numerical methods for solving nonlinear problems MC <u>Lecture 11.</u> Construction of geometry of the calculated region and its sampling using the open CAD-system Gmsh. Software implementation of numerical methods for solving nonlinear problems mcc in the Mathcad system. Conducting numerical experiments in the Mathcad system. Visualization of the results of numerical analysis in an open graphical package for interactive visualization of ParaView. <u>Tasks on the IWS. Literature:</u> [5]: chapter 1, § 1.1–1.3, p. 9–19, section 2, § 2.1–2.3, 20–30, section 3, § 3.1–3.3, 32–42, section 4, § 4.1–4.3, 44–53; electronic lecture notes.	2
Total		18

Practical classes

The main tasks of the cycle of practical classes are the application by graduate students of the theoretical foundations of building physical and mathematical models of the thermal, thermoelectric, thermo-hydrodynamic and thermo-mechanical state of industrial equipment of packaging technology [1–5] to perform the development of numerical models of the physical state, in particular thermal, thermoelectric, thermo- hydrodynamic, mechanical and thermomechanical [A91–A10] using software products Workbench [7], Workbench Fluent [7], OpenFOAM and LIGGGHTS [A11–A13], numerical analysis of the thermal, thermal-hydrodynamic or thermomechanical state of packaging equipment.

No s/p	Name of the topic submitted for self-study	Number of hours of IWS
1	Computer workshop 1. Stationary thermal conductivity of a flat wall, Workbench Fluent - methodical development file - "1ua-ANSYS Academic Teaching example 1.pdf". <u>Tasks on IWS.</u> Mathematical formulation of stationary and non-stationary heat conduction problems. Methods of numerical problem solving. Literature: [1] p. 10–16, p. 81–126; [2] p. 105–106, p. 109–111; [3] p. 8–9, p. 23–84, p. 170–191; [4] p. 9–19. The non-linear non-stationary problem of thermal and electrical conductivity. Mathematical formulation of a nonlinear problem and the method of its numerical solution. Literature: [1] p. 178–180; [2] p. 112–114; [4] p. 20–30.	4
2	Computer workshop 2. Static analysis of structural strength (plate) Workbench - methodical development file - "ANSYS Mechanical static analysis of structural strength (plate) .pdf". <u>Tasks on IWS.</u> Mathematical formulation of the linear problem of static elasticity. The method of numerical solution of the problem. Literature: [1] p. 181–184; [2] p. 106–109; [3] p. 9–18, p. 87–127, p. 193–216. Mathematical formulation of the problem of dynamic elasticity taking into account large deformations and contact interaction of a mechanical nature. Literature: [2] p. 111–112. Mathematical formulation of the coupled problem of thermoelasticity. Literature: [2] p. 114–116. The problem of elastic-plastic behavior of an isotropic material. Mathematical formulation of a nonlinear problem and the method of its numerical solution. Literature: [4] p. 32–42.	4
3	Computer workshop 3. Mixing fluid flows of different temperatures in a pipe tee - Fluid Flow and Heat Transfer in a Mixing Tee, Workbench Fluent -	4

No s/p	Name of the topic submitted for self-study	Number of hours of IWS
	methodological development file - "fluent_13.0_workshop01-mixingtee.pdf", model discretization file - "fluidtee.meshdat» [6]. Calculations of material and heat balances. <u>Tasks on IWS.</u> Mathematical formulation of the problem of thermo-hydrodynamics during laminar flow of an incompressible Newtonian fluid. <u>Literature:</u> [2] p. 117–120. Mathematical formulation of the problem of thermal-hydrodynamics during the turbulent flow of an incompressible Newtonian fluid based on the standard model. <u>Literature:</u> [2] p. 120–122. The method of finite volumes. <u>Literature:</u> [1] p. 127–139. Methods of drawing up energy and heat balances of industrial equipment. <u>Literature:</u> [1] p. 185–190.	
4	Computer workshop 4. Construction of pipe tee geometry and discretization for KP3. Workbench – DesignModeler, Meshing [6]. <u>Tasks on IWS.</u> Physical equations for a non-Newtonian fluid. <u>Literature:</u> [2] p. 123–124. Examples of the application of mathematical modeling for the development of energy-saving structures and regulations for the operation of industrial equipment. <u>Literature:</u> [1] p. 198–227.	4
5	Computer workshop 5. Using the Discrete Phase Model (DPM), Workbench Fluent – methodological development file – "fluent_13.0_workshop06-dpm.pdf", model discretization file – "dpm_tutorial.msh" [6]. <u>Tasks on IWS.</u> Mathematical models of dilatant (composite) substances. Formulation of the thermo-hydrodynamic problem for a liquid of the Bingham-Papanastasiou type. <u>Literature:</u> [2] p. 124–127. Mathematical model of the impregnation process of porous media during the manufacture of composite containers. <u>Literature:</u> [2] p. 127–129.	4
6	Computer workshop 6. Cooling of electronic devices due to natural convection and thermal radiation - Electronics Cooling with Natural Convection and Radiation, Workbench Fluent - methodical development folder - "workshop4-electronics", model discretization file - "heatsink.msh.gz" [6]. Calculations of material and heat balances. <u>Tasks on IWS.</u> Methods of drawing up energy and heat balances of industrial equipment. <u>Literature:</u> [1] p. 191–197.	4
7	Computer workshop 7. Tank flushing - two-phase model. Tank Flushing Example, Workbench Fluent - method file. development - "fluent_13.0_workshop07-tank-flush.pdf", mesh - file "tankflush.msh.gz" [6]. <u>Tasks on IWS.</u> Mathematical formulation of the two-phase flow problem based on the VOF model. Mathematical formulation of the multiphase flow problem based on the Euler model. <u>Literature:</u> [6].	8
8	Computer workshop 8. Determining the angle of a natural slope based on the results of modeling the process of forming a hill of loose material, LIGGGHTS, ParaView [D6–D7]. <u>Tasks on IWS.</u> Formulation of the problem of a moving layer of bulk material. <u>Literature:</u> [2] p. 130–133; [7] p. 33–35, p. 37–43. The problem of elastic-plastic behavior of loose material. Mathematical formulation of a nonlinear problem and the method of its numerical solution. <u>Literature:</u> [4] p. 44–53.	4
Total		36

Laboratory classes (computer workshop)

Laboratory classes are not provided for the working curriculum.

6. Independent work of a graduate student

No s/p	Name of the topic submitted for self-study	Number of hours of IWS
1	Topic 1.1. Basics of tensor calculus <i>Strainers. The rank of tensor. Physical laws. Vectors and scalars. Coordinate systems. Basic vectors. Triedron of single vectors. Actions on vectors. Diads and diadicas. Index designations. The interval for changing indexes and agreeing on the summation. Approval of summation in symbolic designations. Coordinate conversion. General concept of strain. Metric tensor. Cartesian strainers.</i> <u>Literature:</u> [1]: section 1, § 1.1–1.23, pp. 11–52, [A1]: Annex A, § A.1, pp. 319–335.	5
2	Topic 1.2. Tense state <i>The concept of a solid environment. Homogeneity. Isotropics. Density. Massive forces. Surface forces. The principle of tension koshi. Vector of tension. Tense state at the point. Tensor tension. The relationship between the tensor of tension and the vector of tension. Balance of forces and moments. Symmetry of tension tensor. The laws of transformation of the tension tensor. Surfaces of cauchy tension. The main tensions. Tensor tension invariants. Ellipsoid of tension. Maximum and minimum tactile stress. Mora's circles for tension. Flat tense state. Deviator and bullet tensor tension.</i> <u>Literature:</u> section 2, § 2.1–2.14, cc 53–78; [A1]: Annex A, § A.2, pp. 335–344.	5
3	Topic 1.3. Deformed state <i>Particles and points. Solid environment configuration. Deformation and flow. Radius vector. Vector of movement. Lagrangian and Eilers description of movement. Deformation gradients. Move gradients. Strain strains. Strainers of finite deformations. Theory of small deformations. Strainers of infinitesimal deformations. Relative movement. Line rotation tensor. Vector of rotation. Geometric content of tensors of linear deformations. Coefficient of length. Interpretation of finite deformations. Strainers of coefficients of length. Tensor turn. Properties of transformation of deformation tensors. The main deformations. Invariants of deformation. 3-D expansion. Bullet tensor and deviator deformations. Flat deformation. Mora circles for deformation. Compatibility equations for linear deformations.</i> <u>Literature:</u> [1]: section 3, § 3.1–3.16, 79–107; [A1]: Annex A, § A.3, pp. 344–361.	5
4	Topic 1.4. Movement and flow <i>Current. Material derivative. Speed. Acceleration. Instant speed field. Trajectory. Tok lines. Established movement. Deformation rate. Edging. Increase of deformation. Physical interpretation of tensors of the rate of deformation and edging. Material derivatives of the volume element, surface element and linear element. Material derivatives of the integral by volume, integral on the surface and linear integral.</i>	5

No s/p	Name of the topic submitted for self-study	Number of hours of IWS
	<i>Literature:</i> [1]: section 4, § 4.1–4.7, 108–122; [A1]: Annex A, § A.4, pp. 361–368.	
5	Topic 1.5. Laws of solid environment mechanics <i>The law of conservation of mass. Equation of indissolubility. Theorem on the change in the number of movements. Equation of motion. Equilibrium equation. Theorem on the change in the moment of the number of movements. The law of conservation of energy. The first law of thermodynamics. Equation of energy. Equation of state. Entropy. The Second Law of Thermodynamics. Clausius-Duham inequality. Dissipative function. Defining equations. Thermomechanical and mechanical continuums.</i> <i>Literature:</i> [1]: section 5, § 5.1–5.7, 123–134; [A1]: Annex A, § A.5, pp. 368–370.	4
6	Topic 1.6. Linear elasticity theory <i>Hooke's generalized law. The function of deformation energy. Isotropic and anisotropic media. Symmetry of elastic properties. Isotropic environment. Elastic steels. Setting static and dynamic problems of elasticity theory. The superposition theorem. The only solution. The Principle of Saint-Venant. Flat problems of elasticity theory. Flat tense state and flat deformation. The functions of Erie's tension. Two-dimensional static problems of elasticity theory in polar coordinates. Hyperelasticity. Hypoelastcity. Linear thermal elasticity.</i> <i>Literature:</i> [1]: section 6, § 6.1–6.10, 136–154; [A1]: Annex A, § A.6, pp. 370–373.	4
7	Topic 1.7. Mechanics of liquids and gases <i>Fluid pressure. Tensor viscous tensions. Barotropic current. Defining or physical equations. Runoff liquids. Newtonian fluids. The basic equations of Newtonian fluid. The Navier-Stokes-Duham equation. Established current. Hydrostatics. Dead-up current. Perfect liquid. Bernouli equation. Circulation. Potential current. Flat potential current.</i> <i>Literature:</i> [1]: section 7, § 7.1–7.6, 155–165; [A1]: Annex A, § A.7, pp. 373–377.	4
8	Topic 1.8. Theory of Plasticity <i>Basic provisions and definitions. Idealized diagrams of plastic behavior. Conditions of plasticity. Criteria of Tresca and Mises. Space of tension. P-plane. Surface fluidity. Behavior of the material beyond fluidity. Isotropic and kinematic strengthening. The relationship between stresses and deformations in the plastic state. Theory of Plastic Potential. Equivalent tension. Equivalent increase in plastic deformation. Work on plastic deformations. Hypotheses of strengthening. The ratio of the incremental theory of plasticity for the environment with isotropic strengthening. Deformation theory of plasticity. Problems of elastic plasticity.</i> <i>Literature:</i> [1]: section 8, § 8.1–8.13, 172–198; [A1]: Annex A, § A.8, pp. 377–379.	4
9	Topic 1.9. Linear viscoelasticity <i>Viscous behavior of the material. The simplest mechanical models of knitting behavior. Generalized models. Linear differential operator equation. Creep and relaxation. Functions of creep and relaxation. Integrals of heredity. Integrated modules and malleability. Three-</i>	4

No s/p	Name of the topic submitted for self-study	Number of hours of IWS
	<i>dimensional theory of linear viscosity. Analysis of the viscous stress state. Principle of conformity.</i> <u>Literature:</u> [1]: section 9, § 9.1–9.8, 200–217; [A1]: Annex A, § A.9, pp. 380–383.	
10	Topic 1.10. Turbulent movement of a solid medium <i>Classic turbulence models for Reynolds stresses. Prandtl model, built on an algebraic expression. Prandtl-Colmour model with one equation. isotropic turbulence model with two scalar equations. Mathematical models for numerical modeling of turbulent flows. RANS turbulence models. Les turbulence models. Hybrid models of DES turbulence.</i> <u>Literature:</u> [A5]: lecture. 9, p. 80–86; [1]: section 11, § 11.1–11.4, 256–271; [4]: section 9, § 9.1–9.3, 198–218.	4
11	Topic 2.1. Mathematical formulation of nonlinear problems MC <i>Solving the nonlinear problem of thermal conductivity on the example of heating / cooling of the cylinder by finite elements using triangular finite elements in gmsh, mathcad and paraview environments. Comparison of results with precise solutions and numerical solutions obtained using ANSYS software products.</i> <u>Literature:</u> [5]: section 1, § 1.1–1.3, p. 9–19.	5
12	Topic 2.2. Numerical methods for solving nonlinear MCC problems based on the finite element method <i>Solving the nonlinear problem of thermal conductivity on the example of heating a cylindrical conductor using the ITU method using triangular finite elements in gmsh, mathcad and paraview environments. Solving the nonlinear problem of elastic plasticity on the example of a thick-walled cylinder under internal pressure using the ITU method using an implicit reverse display algorithm in Gmsh, Mathcad and ParaView environments. Solving the nonlinear problem of elastic plasticity on the example of a loose material cone using the ITU method using an implicit reverse display algorithm in gmsh, mathcad and paraview environments. Comparison of results with precise solutions and numerical solutions obtained using ANSYS software products.</i> <u>Literature:</u> [5]: section 2, § 2.1–2.3, cc. 20–30, section 3, § 3.1–3.3, 32–42, section 4, § 4.1–4.3, 44–53.	5
13	Topic 2.3. Numerical methods for solving nonlinear problems of the heat-hydrodynamic state of Newtonian and nenuton liquids on the basis of the finite volume method <i>Solving nonlinear problems of laminar and turbulent motion of Newtonian incompressible fluid using the SIMPLE algorithm in OpenFOAM and ParaView environments. Comparison of results with numerical solutions obtained using ANSYS software products.</i> <u>Literature:</u> [4]: section 5, § 5.1–5.2, 127–139, section 9, § 9.1–9.3, 198–218.	5
14	Topic 2.4. Software implementation of numerical methods for solving nonlinear problems MC <i>Solving nonlinear problems of laminar movement of non-Newtonian fluids using OpenFOAM and ParaView method. Comparison of results with numerical solutions obtained using ANSYS software products.</i> <u>Literature:</u> [1]: section 7, § 7.7–7.8, 166–171.	7

No s/p	Name of the topic submitted for self-study	Number of hours of IWS
15	Exam	30
Total		96

Control works

The working curriculum does not provide for control work.

Politics and control

7. Policy of educational component

Violation of the Code of Academic Integrity of the National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute" is a serious violation, even if it is unintentional. The Code is available at the link: <https://kpi.ua/code.3>.

In particular, compliance with the Code of Academic Integrity means that all work in exams and tests must be performed individually. When doing independent work, PhD applicants can consult with teachers and other PhD applicants, but must independently solve tasks, guided by their own knowledge, skills and abilities. References to all resources and sources (e.g. in reports, independent work or presentations) should be clearly defined and properly designed. In case of joint work with other PhD applicants to perform individual tasks, the teacher should indicate the degree of their participation in the work.

Academic integrity: The policies and principles of academic integrity are defined in Section 3 of the Code. Ethical standards: The norms of ethical behavior of students and employees are defined in section 2.

Requirements for PhD applicants:

- attending lecture and practical classes is a mandatory component of the study of the material, the teacher records the presence in the classroom;*
- the teacher uses Google classroom and ZOOM to teach the material of the current lecture, additional resources, practical classes, etc.;*
- at the lecture, the teacher uses his own presentation material; who, at the end of the lecture, teaches at Google classroom in the relevant educational component, where there is a stream of PhD applicants;*
- at the lecture it is forbidden to distract the teacher from submitting material by PhD applicants, all questions, clarifications, etc., applicants put at the end of the lecture at the allotted time;*
- execution of settlement works (computer workshops) takes place during the hours of independent work and is sent to the teacher's e-mail or telegram;*
- in accordance with the "Code of Honor", the design works (computer workshops) and the preparation of answers to theoretical and practical questions of the examination ticket are performed by PhD applicants independently;*
- encouraging points are put up for: active participation in lectures; preparation of scientific publications on the topics of the educational component.*

8. Types of control and rating system for assessing learning outcomes (RS)

Distribution of study time by types of classes and tasks from the credit module according to the working curriculum:

Semester	Training time		Distribution of study hours					Control measures		
	loans	Acad. hours	Lectures	Practice	Labor.	Ind. classes	IWS	MCW	CGW	Semester certification
2	5	150	18	36	–	–	96	–	–	exam

Current control:

express surveys, calculation works (computer workshops).

Calendar control:

The purpose of its implementation is to improve the quality of training of PhD applicants and monitor the implementation of the schedule of the educational process (compliance with the requirements of the syllabus).

Calendar boundary control is carried out twice a semester.

The first control is week 8, the second is week 14.

Semester control: exam

The rating of the applicant for a PhD in the educational component consists of points received for:

express survey at lecture classes, execution of settlement works (computer workshops) during the hours of independent work of PhD applicants.

Rating points system and evaluation criteria.

Rating points r_k :

a) 5 express surveys at lectures:

- full answer 2 points;
- incomplete answer 1 point;
- unsatisfactory answer 0 points.

b) 8 settlement works:

- performing tasks in full 5 points;
- performing tasks in the amount of 80% 4 points;
- tasks in the amount of 60% 3 points.

Encouraging points r_s .

- additional incentive points +2 points.

Meaning R_c – the starting scale of the RS of current success is equal to the sum of the maximum weight points:

$$R_c = \sum r_k = 2 \times 5 + 5 \times 8 = 10 + 40 = 50 \text{ балів.}$$

Terms of admission to semester control: semester rating (30 points).

Examination points R_E = 40 points:

- **task**
 - the right solution 20 points;
 - solution with arithmetic errors up to 16 points;
 - solution with theoretical errors of 12 points or more;
- **each of the two theoretical questions of the ticket**

full answer
incomplete answer

15 points;
5-8 points.

The size of the R-scale of the RS from the credit module is formed as the sum of the maximum points of the current performance of the R_c and exam scores R_E :

$$R = R_c + R_E = 50 + 50 = 100$$

The rating assessment of the applicant from the credit module is formed as the sum of the points of the current academic success – the starting rating $r_c = \sum r_k + \sum r_s$ (rating points and incentive/penalty points) – and examination points r_E :

$$RD = \sum r_k + \sum r_s + \sum r_E$$

Attestation of the applicant is carried out according to the results of the current value of the individual rating in the educational component – R_c . If the value of the RS is not less than 50% of the maximum possible rating - R_c^A at the time of certification (I at. – $R_c^A = 25$, II at. – $R_c^A = 50$), the PhD applicant is considered satisfactorily certified.

Applicants who have scored a rating in the educational component $R_c < 30$ during the semester are required to increase it before the start of the exam session, otherwise they are not allowed to take the exam and have academic debt. Applicants who scored the required number of points during the semester ($R_c \geq 30$ points) are allowed to take the exam.

Table of conformity of rating points on the university scale:

Number of points	Estimate
100-95	Perfectly
94-85	Very good
84-75	Well
74-65	Satisfactory
64-60	Enough
Less 60	Unsatisfactorily
Admission conditions not met	Not allowed

The procedure for appealing the results of control measures:

Applicants have the opportunity to raise any question regarding the control procedure and expect it to be considered in accordance with predetermined procedures. Applicants have the right to challenge the results of control measures, but necessarily reasonably explaining what comments they do not agree with.

9. Additional information on the educational component (educational component)

The list of theoretical questions that are submitted to current, calendar and semester control is given in Campus, Google classroom.

Distance learning: Distance learning in this educational component is allowed on a certain topic, provided that you agree with PhD applicants. If a small number of PhD applicants have the desire (or due to force majeure) to take an online course on a certain topic, the study of the material in this form is allowed, but PhD applicants must perform all the tasks provided for by the syllabus of the educational component.

The assessment of the exam and assessment for control measures by transferring the results of the online course in this educational component is provided only in case of force majeure of PhD applicants.

Some thematic tasks are performed during the independent work of PhD applicants remotely (with the possibility of consulting with the teacher through social networks, e-mail, etc.).

Extracurricular classes: Consultations (individual and group) in this educational component and independent work of PhD applicants can be carried out with prior consent in the scientific laboratory, in the scientific and technical library of the university and / or at home, respectively. The educational material provided for the assimilation of PhD applicants in the process of independent work is submitted for final control along with the educational material studied during classroom training sessions.

The list of questions submitted for semester control

Theoretical questions

1. Describe the deformation theory of plasticity.
2. Describe the concept of an ideal liquid. Write the Bernoulli equation. What is circulation?
3. Describe the physical interpretation of strain rate and vorticity tensors.
4. Analyze the behavior of the material beyond the yield point. What is the essence of isotropic and kinematic hardening? State the hypotheses of isotropic and kinematic hardening.
5. Analyze the models of dilatant (composite) substances. Write down the statement of the problem of the thermal-hydrodynamic state of a liquid of the Bingham-Papanastasiou type.
6. Describe the conditions of plasticity. Analyze the Tresco and Mises criteria and give their physical interpretation.
7. Analyze the defining or physical equations for a liquid. What are Stokes fluids and Newtonian fluids?
8. Describe the initial and current (actual) state of the elementary volume of the continuum.
9. Analyze the basic equations for a non-Newtonian fluid.
10. Describe the concepts of deformation rate, vorticity, deformation increment.
11. Describe the theorem on the change in the amount of motion. Write the equation of motion. Give the equilibrium equation.
12. Analyze the basic equations for a Newtonian fluid. Give the Navier-Stokes-Duhem equation.
13. Describe the concept of trajectory, streamline, steady motion.
14. Analyze the law of conservation of mass. Write down the continuity equation. What is the difference between the Euler form of the continuity equation and the Lagrangian differential form of its notation?
15. Describe the concept of movement, flow. Define material derivative, velocity, acceleration, instantaneous velocity field.
16. Analyze the law of conservation of energy. Give the mathematical form of writing the first law of thermodynamics and give its formulation. Write the energy equation.
17. Analyze the relationship of the incremental theory of plasticity for a medium with isotropic hardening.
18. Analyze how relative displacement, linear rotation tensor, rotation vector are determined?
19. Analyze what the theory of small deformations is based on? What are infinitesimal strain tensors.
20. Analyze the main provisions and definitions of the theory of plasticity. Give idealized diagrams of the plastic behavior of the material and give their physical interpretation.
21. Describe the concept of tensor, Cartesian tensors, rank and order of tensors.
22. Analyze the formulation of the thermal-hydrodynamic state problem based on the turbulence model, taking into account the compressibility of the viscous medium.
23. Analyze the formulas for double scalar, mixed and double vector products of dyads.
24. Describe the equation of state. Define entropy. Describe the second law of thermodynamics.
25. Analyze examples of coordinate systems. Define basis vectors. What is meant by a triad of unit vectors?
26. Analyze the physical meaning of the Clausius-Duhem inequality. Define the dissipative function.

27. Analyze examples of curvilinear coordinate systems. How do they differ from the Cartesian coordinate system?
28. Describe the defining equations. Define thermomechanical and mechanical continuum.
29. Describe the agreement rules for summation in symbolic notation.
30. Analyze the generalized Hooke's law. Give the physical meaning of the strain energy function.
31. Analyze the transformation laws of Cartesian tensors. What is Kronecker's Delta? State the orthogonality conditions.
32. Describe the definition of an isotropic and anisotropic medium. Analyze the symmetry properties of elastic properties?
33. Describe the main values and main directions of symmetric tensors of the second rank. Write the characteristic equation of the tensor.
34. Describe elastic steels and give their physical meaning.
35. Analyze the Gauss-Ostrogradsky theorem.
36. Analyze the statement of static and dynamic problems of the theory of elasticity. Write the Navier-Cauchy equation and the Beltrami-Michell compatibility equation.
37. Describe the hypotheses of a continuous environment.
38. Analyze the superposition theorem. What can explain the unity of the solutions? Formulate the Saint-Venant principle.
39. Describe the stress state at a point. Define stress tensor.
40. Analyze what is the difference between the definitions of the total stress tensor and the total heat flow vector in RANS and LES models?
41. Describe the conditions for the balance of forces and moments. Prove the symmetry of the stress tensor.
42. Analyze the three main groups of mathematical models for describing turbulent flows. What is the averaging of physical quantities according to Reynolds and Favre?
43. Analyze the definition for the maximum and minimum tangential stress. Where are these stresses used?
44. Analyze the formulation of the problem of the thermal-hydrodynamic state of the reacting medium during the combustion of natural gas without preliminary mixing.
45. Describe the hypotheses of the plane stress state and write down the main equations. Охарактеризуйте в'язкопружний напружений стан. Проаналізуйте принцип відповідності.
46. Analyze what are the configurations of a continuous environment? Define the phenomena of deformation and flow.
47. Describe the plane problems of the theory of elasticity. Analyze how plane stress differs from plane strain?
48. Describe hyperelasticity, hypoelasticity.
49. Analyze the differences between the conservation equations for reacting flows and the conventional Navier-Stokes equations for non-reacting media.
50. Describe the physical interpretation of the initial and final configurations of the medium. What are the differences between the Lagrangian and Eulerian descriptions of motion.
51. Analyze problems of linear thermo-elasticity. Write the Duhamel-Neumann equation.
52. Analyze the definitions of strain gradient and displacement gradient, strain tensor and finite strain tensor.
53. Describe the definition of fluid pressure. Write down the formula for determining the viscous stress tensor. What is a barotropic current characterized by?
54. Analyze the formulas for the Cauchy and Green strain tensors, Euler and Almansi tensors of finite strains.
55. Describe the formulation of the thermo-viscous-elastic-plasticity problem.
56. Analyze what is the viscoelastic behavior of a material? Give the simplest mechanical models of the viscoelastic behavior of the medium and reveal their essence.

57. Analyze the problem of elastic-plasticity. Write down the algorithm for solving the plasticity problem using the Euler and Newton methods.

Practical tasks

1. *The task.* Prove that the vector $\mathbf{v} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ is perpendicular to the plane given by the equation $ax + by + cz = \lambda$ (see figure).

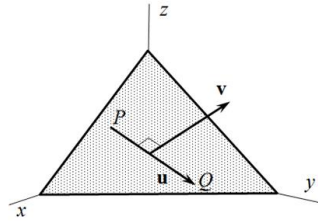


Fig. To the task

2. *The task.* For the displacement field $\mathbf{u} = (x_1 - x_3)^2 \mathbf{e}_1 + (x_2 + x_3)^2 \mathbf{e}_2 - x_1 x_2 \mathbf{e}_3$ subject to the restrictions accepted in the theory of small deformations ($\mathbf{L} = \mathbf{E}$), determine the linear deformation tensor, the linear rotation tensor and the rotation vector at a point $P(0, 2, -1)$.
3. *The task.* Find expressions for creep characteristics in the Kelvin and Maxwell models by direct integration of the equations $\dot{\varepsilon} + \frac{\varepsilon}{\tau} = \frac{\sigma_0 [U(t)]}{\eta}$ i $\dot{\sigma} + \frac{\sigma}{\tau} = G \varepsilon_0 [\delta(t)]$ in accordance.
4. *The task.* Check the correctness of the relationships between stresses and strains represented by the formulas $\dot{\varepsilon} = \frac{\dot{\sigma}}{G} + \frac{\sigma}{\eta}$ i $\sigma = G \varepsilon + \eta \dot{\varepsilon}$ for the Maxwell and Kelvin models, respectively.
5. *The task.* Prove that the Prandtl-Reiss equation $d\varepsilon_{ij}^P = s_{ij} d\lambda$ contains the statement that the principal axes of the plastic strain increment tensor coincide with the principal axes of the stress tensor. Write these equations in terms of principal stresses.
6. *The task.* During loading P in a one-dimensional sample test, the true stress is $\sigma = \frac{P}{A}$, while the technical stress is taken $S = \frac{P}{A_0}$, as where A_0 – is the initial cross-sectional area, and A – is its current value. Find the conditions of the maximum load during plastic deformation with conservation of volume ($A_0 L_0 = AL$).
7. *The task.* Tensor of the second rank $\hat{\mathbf{B}}$ is skew-symmetric, i.e. $\hat{\mathbf{B}} = -\hat{\mathbf{B}}_v$. Show that $\hat{\mathbf{B}}_v \times \mathbf{a} = 2\mathbf{a} \cdot \hat{\mathbf{B}}$.
8. *The task.* Tensors of strain rate and stress are given at some point of the solid medium
- $$D_{ij} = \begin{pmatrix} 1 & 6 & 4 \\ 6 & 3 & 2 \\ 4 & 2 & 5 \end{pmatrix} \text{ i } \sigma_{ij} = \begin{pmatrix} 4 & 0 & -1 \\ 0 & -2 & 7 \\ -1 & 7 & 8 \end{pmatrix}.$$
- Determine the value at this point λ stress power $D_{ij} \sigma_{ij}$.
9. *The task.* Prove the identity
- $$\varepsilon_{pqs} \varepsilon_{mnr} = \begin{vmatrix} \delta_{mp} & \delta_{mq} & \delta_{ms} \\ \delta_{np} & \delta_{nq} & \delta_{ns} \\ \delta_{rp} & \delta_{rq} & \delta_{rs} \end{vmatrix}.$$
10. *The task.* Find the main directions and main values of the Cartesian tensor $\hat{\mathbf{T}}$ of the second rank, which is represented by the matrix

$$[T_{ij}] = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

11. *The task.* Stress vectors $t_i^{(e_1)}$, $t_i^{(e_2)}$, $t_i^{(e_3)}$, acting on three coordinate platforms are given. Show that the sum of the squares of the modules of these vectors does not depend on the orientation of each coordinate plane.
12. *The task.* The stress state at some point is given by the stress tensor

$$\sigma_{ij} = \begin{pmatrix} \sigma & a\sigma & b\sigma \\ a\sigma & \sigma & c\sigma \\ b\sigma & c\sigma & \sigma \end{pmatrix},$$

where a, b, c – constants, σ – some stress value. Define constants a, b, c so that the stress vector on the octahedral site with unit normal $\mathbf{n} = \frac{1}{\sqrt{3}}\mathbf{e}_1 + \frac{1}{\sqrt{3}}\mathbf{e}_2 + \frac{1}{\sqrt{3}}\mathbf{e}_3$ would be zero.

13. *The task.* Write the defining equation for a Newtonian fluid with zero volume viscosity $\chi^* \equiv 0$.
14. *The task.* Determine the average normal stress $\sigma_{ii}/3$ in an incompressible Stokes (nonlinear) fluid, for which $\tau_{ij} = \alpha D_{ij} + \beta D_{ik} D_{kj}$, moreover α, β – constants, де $D_{ij} = \left(\frac{\partial v_j}{\partial x^i} + \frac{\partial v_i}{\partial x^j} \right) = v_{j,i} + v_{i,j}$ – components of the strain rate tensor, s^{-1} ; v_i – components of the velocity vector, m/s.
15. *The task.* Derive the energy equation $kT_{,ii} = \rho c_v \dot{T} + (3\lambda + 2\mu)\alpha T \dot{\epsilon}_{ii}$ for thermo-elasticity, while using the free energy function $f = u - Ts$.
16. *The task.* Derive the Navier equation $\mu u_{i,jj} + (\lambda + \mu) u_{i,jj} + \rho b_i = 0$.
17. *The task.* Show that the stress tensor can be expanded σ_{ij} on the spherical and deviator parts is possible only in one way.
18. *The task.* Relatively combined material X_i and spatial axes x_i solid medium displacement field is specified $x_1 = X_1$, $x_2 = X_2 + AX_3$, $x_3 = X_3 + AX_2$, where A – constant value. Determine the components of the displacement vector in material and spatial form (that is, in Lagrangian and Eulerian variables).
19. *The task.* A displacement vector is set relative to the combined material and spatial axes $\mathbf{u} = 4X_1^2\mathbf{e}_1 + X_2X_3^2\mathbf{e}_2 + X_1X_3^2\mathbf{e}_3$. Determine the shifted position of the particle that was initially located at the point $(1, 0, 2)$.
20. *The task.* Some volume of the continuous medium undergoes deformation $x_1 = X_1$, $x_2 = X_2 + AX_3$, $x_3 = X_3 + AX_2$, where A – constant value. Calculate the Green's strain tensor $\hat{\mathbf{G}}$ and use it to determine the Lagrangian tensor of finite deformations $\hat{\mathbf{L}}_G$.
21. *The task.* The velocity field is given $v_1 = x_1/(1+t)$, $v_2 = 2x_2/(1+t)$, $v_3 = 3x_3/(1+t)$. Find the components of the acceleration vector.
22. *The task.* The velocity field is given by a vector $\mathbf{v} = x_1^2 t \mathbf{e}_1 + x_2 t^2 \mathbf{e}_2 + x_1 x_3 t \mathbf{e}_3$. Determine the speed and acceleration of the particle at the moment $t=1$ at the point $P(1, 3, 2)$.
23. *The task.* Some flow is given by the velocity field $v_1 = 0$, $v_2 = A(x_1 x_2 - x_3^2) e^{-Bt}$, $v_3 = A(x_2^2 - x_1 x_3) e^{-Bt}$, де A и B – constants. Find the velocity gradient $\partial v_i / \partial x_j$ for this movement and calculate the strain rate tensor $\hat{\mathbf{D}}$ and the vorticity tensor $\hat{\mathbf{V}}$ at the point $P(1, 0, 3)$ in a moment of time $t = 0$.
24. *The task.* For rotation of a perfectly rigid body with speed $\mathbf{v} = 3x_3\mathbf{e}_1 - 4x_3\mathbf{e}_2 + (4x_2 - 3x_1)\mathbf{e}_3$ determine the velocity vortex vector $\boldsymbol{\Omega}$ and show that $\mathbf{v} = \boldsymbol{\Omega} \times \mathbf{x}$.

25. *The task.* Довести, що $d(\ln J)/dt = \operatorname{div} \mathbf{v}$.

26. *The task.* Prove that the Lagrangian form $\frac{d}{dt}(\rho J) = 0$ continuity equation and its Euler form

$$\frac{d\rho}{dt} + \rho v_{k,k} = 0$$
 are equivalent.

27. *The task.* The theorem on the change of the amount of motion in the differential form is expressed

$$\text{by the equation } \frac{\partial(\rho v_i)}{\partial t} = \rho b_i + (\sigma_{ij} - \rho v_i v_j)_{,j}.$$

Prove that the equation of motion $\sigma_{ji,j} + \rho b_i = \rho \dot{v}_i$ follows from this equation.

28. *The task.* Prove that the strain energy density u^* for an isotropic sound medium can be expressed through the strain tensor in the form $u^* = \lambda (\operatorname{tr} \hat{\epsilon})^2 / 2 + \mu \hat{\epsilon}^2 : \hat{\epsilon}$ and through the stress tensor in the form $u^* = [(1 + \nu) \hat{\sigma} : \hat{\sigma} - \nu (\operatorname{tr} \hat{\sigma})^2] / (2E)$.

29. *The task.* There is a state of all-round uniform compression with a stress tensor $\sigma_{ij} = -p \delta_{ij}$. Get formulas $K = \frac{E}{3(1-2\nu)}$, $K = \frac{3\lambda + 2\mu}{3}$ for volumetric compressibility module (ratio of pressure to volume change).

30. *The task.* Perform the rotation operation of Hooke's law for stress due to deformation $\sigma_{ij} = \lambda \delta_{ij} \epsilon_{kk} + 2\mu \epsilon_{ij}$, to obtain the relationship for strain due to stress

$$\epsilon_{ij} = \frac{-\lambda}{2\mu(3\lambda + 2\mu)} \delta_{ij} \sigma_{kk} + \frac{1}{2\mu} \sigma_{ij}.$$

Work program of the educational component (syllabus):

Compiled by Professor, Doctor of Technical Sciences, Professor, Anton Karvatskyi

Approved by the Department of Chemical, Polymeric and Silicate Engineering (Protocol No. 17 of 04.06.2025)

Approved by the Faculty Methodical Commission (Protocol No. 11 of 27.06.2025)